Kinetic maximal L^2-regularity for nonlocal Kolmogorov equation and application

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We study the linear and nonlinear nonlocal abstract Kolmogorov equations. The equations includes the abstract operator A in a Banach space E. Here, the kinetic maximal L^2-regularity for the linear equatıon is derived in terms of E-valued Sobolev spaces. Moreover, we show that the solution u is also regular in time and space variables when u is assumed to have a certain amount of regularity in velocity. Finally, the kinetic maximal L^2-regularity for the linear equation can be used to obtain local existence and uniqueness of solutions to a quasilinear nonlocal Kolmogorov type kinetic equation. We first, consider the Cauchy problem for Kolmogorov type nonlocal kinetic linear equation

$$\partial_t u + \upsilon.\nabla_x u + a * (-\Delta_\upsilon)^{\frac{\beta}{2}} u - Au = f, \tag{1.1}$$

$$u(x,\upsilon,0) = 0,$$

$$x = (x_1, x_2, ..., x_n), \ \upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n) \in \mathbb{R}^n, \ t \in \mathbb{R},$$

where A is a linear operator in a Banach space E, $a = a(x,\upsilon)$ is a complex valued function, $a * u$, denotes the convolution of a, u and $\beta \in (\,0,2\,]$. The variable x describes the position, and the variable υ describes the velocity. Here, $u = u(x,\upsilon,t)$ is an unknown, $f = f(x,\upsilon,t)$ is a given E-valued functions and

$$\upsilon.\nabla_x u = \sum_{k=1}^{n} \upsilon_k \frac{\partial u}{\partial x_k}.$$

We show the kinetic maximal L^p-regularity of the problem (1.1) under some assumptions on the function a and linear operator A. This fact can be used to obtain the existence, uniqueness and regularity properties of the corresponding nonlinear kinetic equation

$$\partial_t u + \upsilon.\nabla_x u + a * (-\Delta_\upsilon)^{\frac{\beta}{2}} u - Au = F(x,\upsilon,t,u) + f, \tag{1.2}$$

where F is a nonlinear operator in E and $f = f(x,\upsilon,t)$ is a given E-valued function.